CURVED FOOTBRIDGES SUPPORTED BY A SHELL OBTAINED AS AN ENVELOPE OF THRUST-LINES

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SUMMARY

Since Maillart's curved arch bridges in concrete, different typologies of curved bridges in both steel and concrete have been constructed. On the contrary, since Musmeci's shell supported bridge in Potenza, few shell bridges have been designed. This paper show how to design a curved shell supported footbridge shaping the shell by using the Thrust Network Analysis. As a matter of fact, a shell supported footbridge is shaped between two curved arches, the one supporting the deck and with assighed layout depending on the layout of the road, and the one that defines the shell boundary at the lower part of the shell, the latter being found as a thrust line in the three-dimensional space. The influence of the boundary conditions on the bridge shape obtained as an envelope of thrust-lines is also investigated.

Keywords: *Shell footbridges, curved bridges, cantilevered deck, ring box girder, externally prestressed, anticlastic shell, concrete, unwished bending, force diagram, thrust network.*

1. INTRODUCTION

When designing bridges with the deck supported by a concrete shell, tensile stresses are to be much lower than compressive stresses, since the former are limited by a much lower tensile strength [1-2]. Obtaining a shell compressed in any direction is therefore the design objective. Shells with anticlastic surface are suitable for this aim, because compressions induced in one direction induce in turn compressive stresses also in the other directions [3]. To shape anticlastic shells, different design methods are available. Among these, it is worth mentioning Force Density method, Dynamic Relaxation method, Particle Spring method, and Thrust Network Analysis (TNA) [4]. With all these methods, the shell form-finding is carried out on an auxiliary net structure (made of cables when in tension or of struts when in compression, as in TNA) whose nodes will define a sufficient number of points that allows to interpolate the shell surface.

After defining the force density as the ratio between force in a bar and its stressed length, and after having assigned the position of the fixed points, in the Force Density method [5] the static equilibrium equations of the nodes (loaded by the external loads) of a discrete network (a grid made of compression bars or a cable net made of tension cables) are first defined, that are then solved through linearizing the system without needing any iterative procedure, thus obtaining the actual position of the nodes, and therefore the form of the network. Dynamic Relaxation is instead an iterative form-finding method that step by step traces the motion of the nodes of a discrete network through small time increments Δt under applied loads until all the nodes (and therefore the whole structure) converge to static equilibrium (with their kinetic energy tending to zero) and tend to assume their final position, thus describing the form of the network structure [6]. The Particle Spring method discretizes surfaces into point nodes (with lumped masses) and lines consisting in springs connecting the nodes, whose stiffness represents in some way that of the shell strip across the spring. Upon applying forces, each node has a number of degrees of freedom between 0 and 3, corresponding to the number of orthogonal reaction forces (between 3 and 0). The equilibrium of the applied external forces on each node and the internal forces in the connecting springs must be satisfied. Equilibrium of each node and shell form are iteratively found [7]. The form finding method herein applied for the form finding of the shell supporting the deck of a curved footbridge is Thrust Network Analysis (TNA), that is the three-dimensional version of Thrust Line Analysis. While the latter applies to two-dimensional structures as arches and barrel vaults, the former applies to three-dimensional vaults, thus allowing to contemporarily obtain a three-dimensional thrust network and find the form of the vault for which only compressions in any direction are allowed [8-9].

An anticlastic shell was chosen to support a cantilevered steel deck, with cantilever I beams fixed at a steel curved box girder continuously supported by the top edge of the concrete shell. The transverse deflections of the cantilever deck (spanning 4 m from the girder) were reduced to acceptable values by externally prestressing the ring box girder, thus inducing a torsional moment counteracting that induced in the box girder by the cantilever I beams of the deck, and therefore reducing its transverse deflections.

2. HISTORICAL BACKGROUND

2.1. Curved bridges

Maillart's Schwandbach Bridge near Hinterfultigen was the first curved arch bridge made of concrete, dating 1933 [10]. Since Mallart's curved bridges in concrete, only few curved bridges were built, while a great impetus in constructing them came from Schlaich's work in the '80s, as well as on his studies on their conceptual design, wonderfully described by some simple but very significant structural schemes on their working principles in [11]. In particular, his studies on prestressing the deck of curved bridges to favourably exploit its curved layout to obtain torsional moments counteracting the ones induced by the applied loads are a milestone in the history of curved bridges. Besides his first curved suspended bridge in Kelheim [11], over the canal between Rheine and Danube, where the deck was a prestressed ring girder made of reinforced concrete, Schlaich developed his suspended and cable-stayed curved footbridges where stresses, instead of flowing in the "black box" of the concrete deck [12], were channelled through structural elements specifically designed to bear them. It is worth comparing the static functioning of the decks of two Schlaich's beautiful curved suspended footbridges, Bochum Bridge [11] and Reddy River Bridge [11], suspended respectively at the inner and the outer deck edge. In Bochum Bridge (Fig. 1a), the axial force in the lower chord of the curved girder (with curvature radius R) and the horizontal projection of the axial force in the hanger can provide a sufficient counter-moment (with lever arm y) to the external load lifted (with lever arm e to the hanger restraint) without no aid of a prestress system applied to the top of the ring girder. Prestressing the deck girder at its top would have provided a counter-moment increase, that would have been necessary if higher external loads were applied.

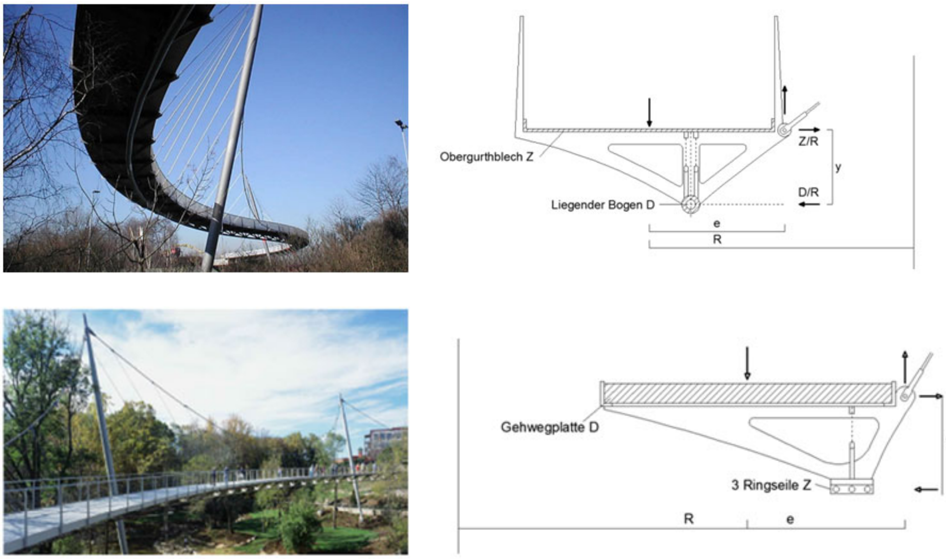

Fig. 1. *Suspended curved footbridge over Gahlensche Strasse in Bochum, Germany (top), and over Reedy River in Greenville, SC, USA (bottom), and details of their deck cross sections.*

Owing to the deck layout and the landscape view, the deck of the curved suspended bridge over the Reddy River in Greenville, S.C., USA (Fig.1b) is instead suspended at the outer edge of the curved girder, so that the direction of the deck curvature radius R is opposite with respect to that of the Bochum Bridge. The inversion of the direction of the flow of the internal forces with respect to the Bochum Bridge (instead suspended at its inner edge) is caused by the suspension of the deck at the outer edge. While tensile forces (provided by three ring tension cables positioned at the girder bottom, see Fig.1b) flow at the intrados of the ring girder, compressions flow in the upper concrete deck. In the transverse direction, a counter-moment (with lever arm y) to the external load lifted (with lever arm e to the hanger restraint) is then provided by the ring shape of the girder, thus reducing transverse deflections of the cantilevered deck.

2.2. Shell supported bridges

Masters in structural architecture, from Frei Otto [13] to Heinz Isler [14] until Sergio Musmeci [15], shaped their concrete shells through auxiliary tension structures with same shape, thus obtaining, for same boundary conditions and self-weight loading, compressed concrete shells with same internal forces and constraint reactions but with the opposite sign. In so doing, for tensile stresses in the auxiliary tension structure also isotropic (as in isotropic materials), a shell of minimal surface is obtained, too.

Fig. 2. *Musmeci's shell bridge over the Basento River in Potenza, Italy.*

Recently, Jiri Strasky proposed two shell supported bridges [16], but the only constructed bridge supported by a concrete shell with minimal (or almost minimal, depending on the form finding procedure) surface, is still the Basento Bridge in Potenza, Italy (Fig. 3), Musmeci's masterpiece designed in the 60's of past century [15].

It is worth comparing the structural functioning of Maillart's bridges [10] and of Musmeci's bridge [1], the former being the first type of stone (artificial) arch bridge with no infill over the arch. In Musmeci's bridge, the typical mechanism of Mallart's bridges of transferring deck loads to the abutments through a certain number of vertical walls and a shell arch can be carried out through a continuous anticlastic shell of almost minimal surface, thus preventing undesired bending effects due to the insertion of vertical walls as in the shell arch of Maillart's bridges.

Fenu et al. [18] compared the structural behaviour of a bridge with deck supported by an anticlastic shell compressed in any direction with a Maillart's bridge with same span and rise (the Traubach Bridge near Habkern [10]). Also, Briseghella et al. applied topology optimization to both arch and shell supported bridges for reducing the area of shell regions where unwished tensile stresses arise [19-21].

3. SHELL FORM FINDING THROUGH THRUST NETWORK ANALYSIS

In the following, the way of applying Thrust Network Analysis (TNA) to shape the shell supporting the curved deck of a shell supported footbridge is illustrated. TNA is a form finding method for designing shell structures compressed in any direction as an envelope of thrust lines. The method was developed (and is particularly suitable) for designing masonry vaults, that must be compressed for the low tensile strength of masonry. TNA is independent of material properties, and is based on "Graphic Statics".

Two types of diagram are used for this aim: the form diagram $\mathbf{\Gamma}$, that is the horizontal projection of the three-dimensional network $\mathbf{G}$ and the force diagram $\mathbf{\Gamma^*}$, that is constituted by the horizontal components of the forces applied on each strut of the compressed network. Form and force diagram are reciprocally related, because each node of $\mathbf{\Gamma}$ corresponds to a polygon of $\mathbf{\Gamma^*}$ with side number equal to the number of struts converging in the corresponding node of $\mathbf{\Gamma}$, and vice versa. Moreover, each side of each polygon of $\mathbf{\Gamma}$ is parallel to a corresponding force of the related equilibrium polygon in $\mathbf{\Gamma^*}$. The magnitude of each force of $\mathbf{\Gamma^*}$ (multiplied by a scale factor ς) represents in $\mathbf{\Gamma}$ the magnitude of the horizontal projection of the axial force in the corresponding strut of the compressed network. Reciprocity conditions between $\mathbf{\Gamma}$ and $\mathbf{\Gamma^*}$ cannot (by themselves) guarantee that all the rods of the network are compressed, because also the further condition that the vectors of all the closed polygons of $\mathbf{\Gamma}$ must rotate counterclockwise with respect to any point inside the closed polygon must be satisfied. Since TNA generates a funicular network of struts where only vertical loads are considered, the equilibrium of the horizontal force components is only to be considered and computed independently of the chosen external vertical loading.

When more than three vectors converge in each node of **Γ**, the network is statically indeterminate (meaning that for a given form diagram many force diagrams exist), but when the designer chooses a certain **Γ*** among all these feasible ones (for instance with compressions always higher than a given value), for given both **Γ** and **Γ*** only one compressed network **G** is identified. Also, all the polygons of **Γ** and **Γ*** must be convex to avoid of obtaining tension forces, that is to prevent tensile stresses in any shell regions.

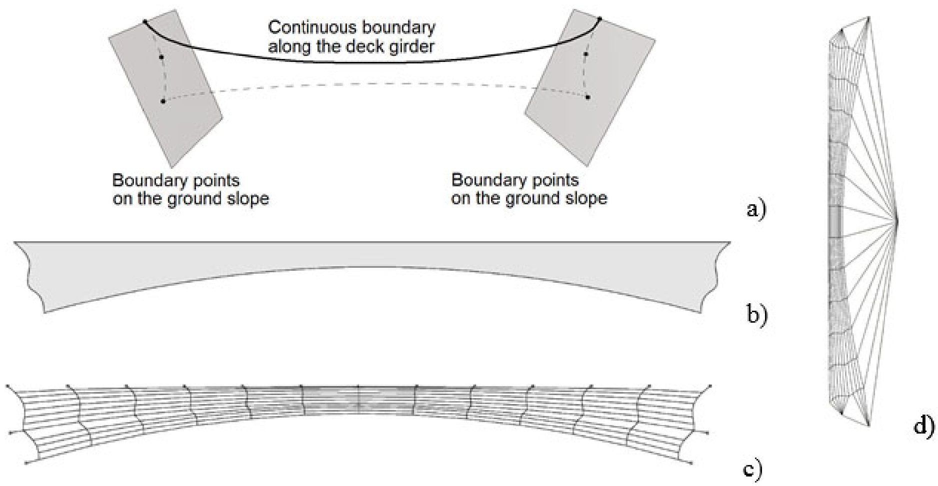

Fig. 3. *a) Boundary conditions, b) starting surface, c) initial form and d) force diagrams.*

TNA can shape compressed networks of any typology. The above graphic procedure can be also implemented analytically, as extensively described in [4].

3.1. Applying TNA to find the form of the shell supporting a curved bridge deck

The design problem of finding the form of a compressed shell supporting the curved deck of a footbridge is faced. An anticlastic shell is particularly suitable to this aim. The design of an anticlastic shell needs suitable boundary conditions. Since the considered shell has to continuously support a deck with given curved layout, the curvature of the parallel of the shell along its top edge must coincide with the deck curvature. Given this boundary condition, an anticlastic shell can be obtained through suitable boundary conditions at the bridge abutments along the ground slope (Fig. 3a).

Using TNA to find the shell form of a shell supported footbridge [11], it must be taken into account that TNA can govern only the form finding of shells whose projection of any shell region on the horizontal plane is not superposed to the projection of another shell region. The shell of the footbridge under consideration does not respect this condition, so that it could seem that TNA cannot be used to shape it. Fortunately, the properties of anticlastic shells can be exploited for this aim, since an important property of anticlastic shells is that if compressions in one direction are increased, compressions in the other directions are also increased. Therefore, the footbridge shell was shaped by starting from its projection on the vertical plane, with no superposition of the projection of different shell regions. Form finding through TNA can be thus performed, even if the equilibrium of horizontal forces in TNA is referred to a vertical plane in the reality and, after achieving it, the equilibrium in the vertical direction in TNA is achieved by imposing the given boundary conditions along the given layout of the curved deck, that in the reality instead lies in a horizontal plane. Nevertheless the procedure leads to good results, as shown below through FEM analysis of the shell, and according to both the fact that in anticlastic shells compressions in one direction result in compressions also in the other directions, and to the fact that, contrary to sinclastic membranes, the form of anticlastic shells is only slightly affected by distributed self-loads.

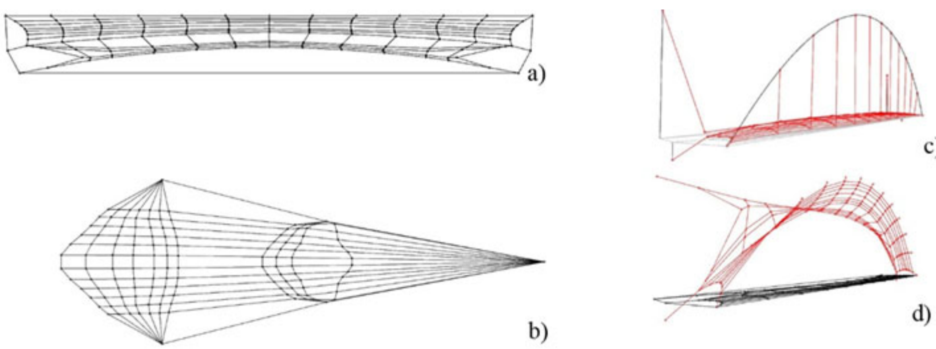

Fig. 4. *a) Final form and b) force diagrams, c) 3D boundary conditions and d) obtained network.*

Therefore, to apply TNA in the form finding problem under consideration, the starting surface was defined by the vertical projection of the boundary points of the shell of the real bridge, with the point boundaries along the abutments connected one another by plane splines (Fig. 3b). The starting surface was then considered as horizontal, and an initial form diagram Γ with quadrilateral polygons was defined (Fig. 3c). A force diagram $\Gamma^*$was then drawn then from the equilibrium of the horizontal thrusts (Fig. 3d), thus obtaining closed quadrilateral force polygons that should represent the equilibrium in the horizontal plane of each node of the Γ. The equilibrium of the forces of these polygons is satisfied since they are closed, but they do not exactly represent the equilibrium of the nodes of Γ, because the direction of the forces of each force polygon deviates from the condition of being parallel to the related vectors in Γ, meaning that these equilibrium polygons do not still represent the horizontal equilibrium of each node of Γ. Through successive iterations, the above condition of parallelism (with a tolerance less than 1%) was finally achieved, with final Γ (Fig. 4a) and Γ^* (Fig. 4b) that satisfied the parallelism condition, and therefore the horizontal equilibrium of the network..

Fig. 4c shows the boundary points in the 3D space. By then stretching the planar form diagram to satisfy the boundary conditions not only in the horizontal plane but also in the 3D space for the 3-rd vertical coordinate, the form of the 3D network was finally found (Fig. 4d). It can be more or less relaxed, depending on the magnitude of the forces channelled in the compressed members of the network, but the equilibrium in the vertical direction of all the 3D networks (more or less relaxed) is always satisfied, provided that the boundary conditions are satisfied. After choosing the most suitable value of membrane relaxation at the end of the TNA procedure, the shape of the shell (thickness 30 cm) minimizing tensile stresses throughout the shell was chosen (Fig. 5).

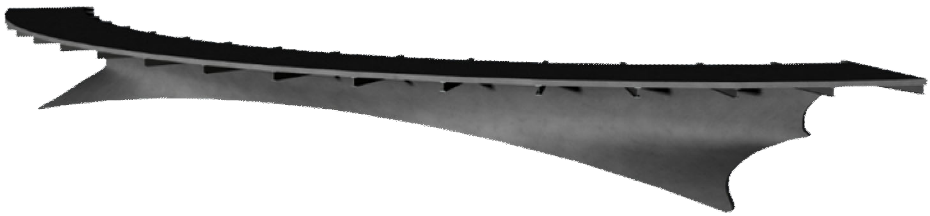

Fig. 5. *Perspective view of the curved shell supported footbridge with shell form found using TNA.*

It is worth noting that, since the shell must be anticlastic, the shell curvature in the vertical direction (that is of the meridian) along the boundary of the given layout of the curved deck must be of the opposite sign with respect to the curvature of the deck (that is of the parallel). After finishing the form finding procedure, the 3D network was finally rotated to its real position with the curved boundary line of its top edge lying in the horizontal plane. Although TNA was applied in an unconventional way (with the shell lateral projection used for constructing the horizontal form diagram), the obtained shell was shown to have Gaussian curvatures always negative practically along the whole shell, thanks to the properties of anticlastic membranes (see Fig. 6). This means that under self-weight loading the shell resulted compressed in any direction, as shown by FEM analysis. Nevertheless, the experience in designing these shell supported bridges has shown that the behaviour of shells with some small central regions with low curvature values was better than that of shells with optimum Gaussian curvature distribution when used to support the cantilevered deck of the shell supported footbridges under consideration. In fact, both external loads applied through a cantilevered deck and prestressing applied to provide a torsional counter moment to the external load lifted caused the arising of unwished tensile stresses in some shell regions, actually not always minimized in shells with optimum Gaussian curvature distribution.

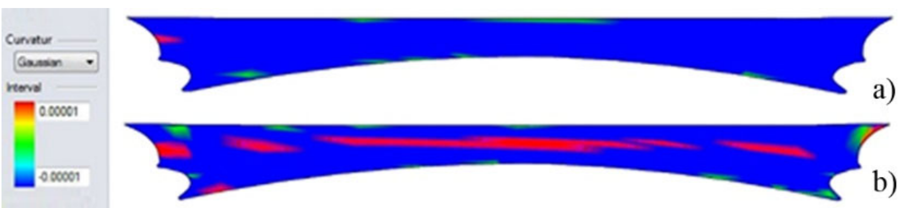

Fig. 6. *a) Best and b) adopted distribution of Gaussian curvatures of the shell.*

4. ARRANGEMENT OF THE CURVED DECK

Curvature and radius (in the horizontal direction) of the concrete shell (grade C 35/42) along the deck were, respectively, 0.0035 m^{-1} and 28.3 m. The length of the related circle chord was 40 m. Hence, the length of the curved deck was 44.48 m. The cantilever deck was supported by a ring box girder (depth 55 cm, breadth 50 cm, thickness 20 mm, three webs, steel grade S355) lying on the top edge of the shell. Girder bottom flange and shell top edge were hinged with one line steel studs spaced 50 cm (Fig. 7).

The ring box girder was transversally stiffened over the studs by steel ribs spaced 50 cm, and made of 16 straight girder segments, each one 2.78 m in length, connected one another through butt-welded joints. The cantilevered deck was made up of cantilever I beams spaced 2.78 m with depth varying between 55 cm (at the end connection with the ring girder) and 32 cm (at the opposite end). They supported a concrete slab (thickness 20 cm, cast on a profiled steel sheet) connected by steel studs to their top flange. Straight girder segments (length 2.78 m) were connected one another to form the ring girder.

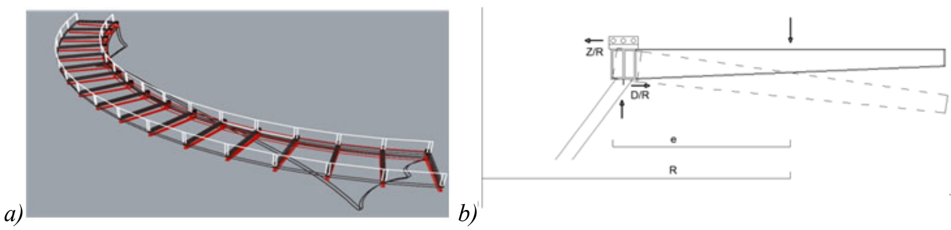

***Fig.** 7. a) Structural scheme of the deck and b) its cross-section.*

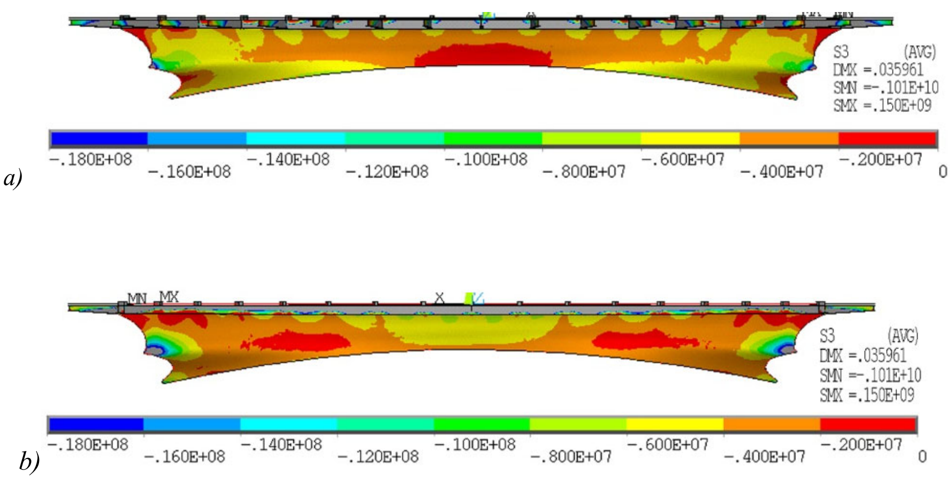

***Fig. 8.** Compressive stresses throughout the shell surface: a) front view and b) back view.*

An external pretensioning system applied an eccentric prestress force at the top flange of the ring girder to induce a longitudinal bending moment in turn inducing, for the intrinsic characteristics of ring girders, a torsional moment allowing to highly reduce transverse deflections of the cantilever deck (Fig. 6b). Three parallel tension cables (f_{ptk} = 1860 MPa, centroid distant 117.5 mm from the girder top), provided an eccentric axial force to the girder (Fig. 7), and were anchored to anchorage heads placed on the joint connecting the girder segments. The cross-sectional area of each cable was 2215 mm^2.

5. STRUCTURAL BEHAVIOUR OF THE CURVED SHELL FOOTBRIDGE

A FEM model of the curved shell footbridge was implemented. Concrete shell, ring box girder and I beams were modelled with 4-node shell elements, while the anchorage heads of the external prestress cables were modelled with 8-node solid elements. The average side of all elements was 5 cm. Link elements were used to model the tension cables.

Besides permanent loads, crowd loads of 5 kN/m^2 were applied, with five load cases considered: 1) full bridge full width, 2) full bridge half width by the outer edge side, 3) full bridge half width by the inner edge side, 4) half bridge full width, 5) two diagonal areas of half width.

The shell was prevalently in compression, with maximum compressive stress 18 MPa (Fig. 8), thus lower than the allowable strength of modern concrete. Tensile stresses resulted minimised by the chosen shell shape (Fig. 9), and limited to 2.2 MPa in the load case 3, and 2 MPa in the load case 1, even if this value was reached only in a small shell region throughout the shell (except local stress concentration close to point restraints). First principal stress was much lower than 1 MPa elsewhere, and lower than 0 (compression) throughout more than half shell surface. Load case 1 was the most critical for stress level of the steel box girder, even if continuously supported by the shell. It was especially stressed at its ends by the prestress force transmitted by the anchorage heads of the pretensioning system.

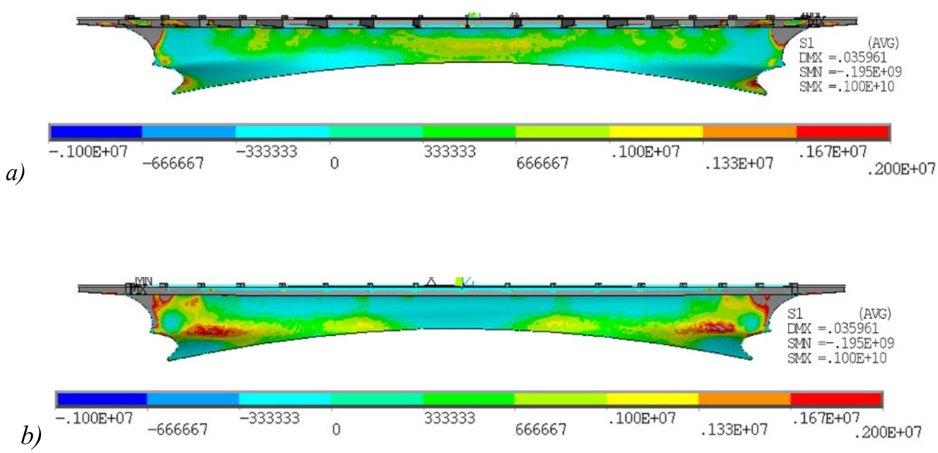

Fig. 9. *Tensile stresses throughout the shell surface: a) front view and b) back view.*

While thickness of the inner segments of the box girder was 20 mm, the end segments needed to be thicker (40 mm), owing to stress concentration close to the anchorage head of the tension cables at the girder ends. On the contrary, in the inner segments the lower stress level allowed the use of steel grade lower than S 355. The force in each cable was 2500 kN (namely a total force of 7500 kN) corresponding to a tensile stress in the cables of 1130 MPa. This prestress level was achieved through an initial strain of 0.0053.

Maximum deflection was 35.9 mm, and occurred at mid-span in the load case 1 (Fig. 10). The shell footbridge was therefore very stiff, with a span length over deflection ratio of 40000 / 35.9 = 1113. Prestressing the ring box girder induced a torsional counter moment that allowed to limit transverse deflections in the cantilevered deck: in particular, the maximum difference of deck deflections in the transverse direction was 19.3 mm. The ratio of double cantilever length over deck transverse deflection was limited to 413.

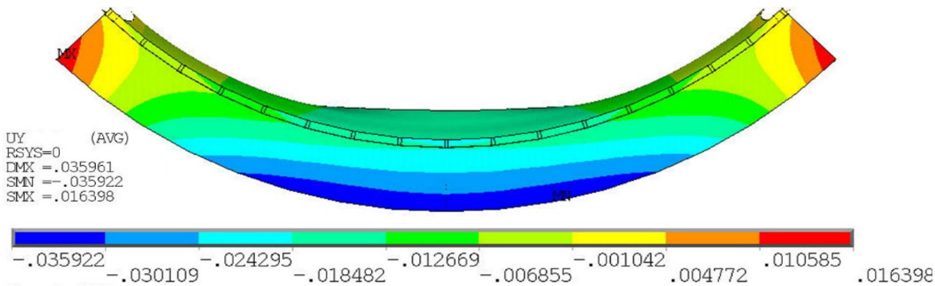

Fig. 10. *Contour plot of the vertical displacements.*

6. CONCLUSIONS

The anticlastic shell of a curved shell supported footbridge was unconventionally shaped using Thrust Network Analysis because a horizontal starting surface could not be obtained as shell horizontal projection without superposition of some shell projected points. Accounting for the intrinsic characteristics of anticlastic membranes, for which compressions in one direction result in compressions in any direction, the shell was shaped through projection in the real vertical plane considered in TNA as a horizontal plane where to obtain the 2D equilibrium of the network. After finding the 3D network, the footbridge shell was interpolated, and good structural behaviour was observed.

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REFERENCES

[1] FENU L., MADAMA G., On a method of shaping R/C shells with heuristic algorithms and based on Musmeci's work. *Milan Technical University, Studi e Ricerche / Studies and Researches*. 2005; 25, pp. 199-238.

[2] MUSMECI S., *La statica e le strutture*, Roma: Edizioni Cremonese, 1971 (in Italian).

[3] FLUGGE W., *Stresses in shells,* Berlin, Heidelberg: Springer; 1st edition, 1960.

[4] ADRIAENSSENS S., BLOCK P., VEENENDAAL D., WILLIAMS C. (Edited by), *Shell structures for Architecture. Form finding and optimization.* New York: Routledge, 2014.

[5] SCECK H.J., The Force Density method for form finding and computation of general networks. *Comput. Methods in Appl. Mech. Eng.* 1974; 3, pp. 115-134.

[6] DAY A.S., An introduction to Dynamic Relaxation, *The Engineer.* 1965.

[7] KILIAN A., OCHSENDORF J., Particle Spring Systems for structural form finding, *Journal of the International Association for Shell and Spatial Structures*, 46 (2), 2005, pp. 77-84.

[8] BLOCK P., *Thrust Network Analysis: exploring three-dimensional equilibrium.* PhD Dissertation (Advisor: John A. Ochsendorf), Massachusetts Institute of Technology. Boston, USA, 1999.

[9] RIPPMANN M., LACHAUER L., BLOCK P., Interactive vault design. *International Journal of Space Structures*, 2012, 27(4), pp. 219-230.

[10] BILL M., *Robert Maillart.* Zurich: Girsberger; 1955.

[11] SCHLAICH J., BERGERMANN R., *Leicht Weit (Light Structures)*, Munchen, Prestel Verlag, 2003.

[12] SCHLAICH, J., SCHÄFER K., JENNEWEIN M., Toward a consistent design of structural concrete. *PCI Journal, Special Report*, 1987, 32(3).

[13] OTTO F., *Tensile structures*, Cambridge, MA: The MIT Press, 1973.

[14] ISLER H., Concrete shells derived from experimental shape. *Structural Engineering International*,1994, 4(3).

[15] NICOLETTI M., *Sergio Musmeci: Organicità di forme e forze nello spazio.* Turin: Testo & Immagine,, 1999 (in Italian).

[16] STRASKY J., HUSTY I. and PARTNERS., http://www.shp.eu/en/strasky-husty-a-partneri/projects/research/

[17] FENU L., MADAMA G., TATTONI S., On the conceptual design of r/c footbridges with deck supported by shells of minimal surface. *Milan Technical University, Studi e Ricerche / Studies and Researches*. 2006; 26, pp. 103-126.

[18] BRISEGHELLA B., FENU L., LAN C., MAZZAROLO E., ZORDAN T., Application of topological optimization to bridge design. *Journal of Bridge Engineering (ASCE).* 2013; 18(8) , pp. 790-800.

[19] BRISEGHELLA B., FENU L., FENG Y., MAZZAROLO E., ZORDAN T., Topology optimization of bridges supported by a concrete shell. *Structural Engineering International*. 2013; 23(3), pp. 286-294.

[20] BRISEGHELLA B., FENU L., FENG Y., MAZZAROLO E., ZORDAN T., Optimization indexes to identify the optimal design solution of shell-supported bridges, *Journal of Bridge Engineering (ASCE).* 2016, 21(3), Art. n. 04015067.