ANALYSIS OF ARCH BRIDGES WITH FINITE COMPRESSION STRENGTH MATERIAL UNDER HORIZONTAL LOADINGS

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SUMMARY

The mechanism method, which is based on the assumption that stone arches fail by forming pin joints, is certainly very suitable for the evaluation of safety of stone and masonry arch bridges. Several studies have been done to analyse the limit behaviour under dead - plus - variable loads. In this paper the method is applied to arch bridges subject to horizontal acceleration by removing the most limiting hypothesis, i.e., the infinite compression strength of the material. This is supposed to have a rigid-perfect plastic behaviour in compression and no tension strength. A numerical investigation was carried out, which allowed pointing out the main aspect of the structural behaviour and the influence of the various parameters on the horizontal acceleration for which the structure is turned into a mechanism.

Keywords: *Arch Bridges, Mechanism method, Seismic analysis.*

1. INTRODUCTION

The limit analysis is certainly very suitable for the evaluation of safety of stone and masonry arch bridges. It is based on the assumption that stone arches fail by forming pin joints, as demonstrated by old but also by more recent experimental studies; as a result, the collapse must be viewed as a geometrical issue rather than a problem of strength of material. Numerical solutions based on the mechanism method were proposed by different authors and were applied to study the limit behaviour and to find the collapse mechanisms of stone arches under dead - plus - vertical live loads or horizontal forces simulating the seismic actions.

The evaluation of safety of old masonry arch bridges under seismic loads is of interest for bridge owners and quick methods of analysis are certainly useful, especially in preliminary approaches to structural analysis, whereas finite element [1] or discrete element methods [2] are today the preferred way for in-depth investigations.

In the field of expeditious methods, the ones based on the limit analysis (the so called mechanism method) are widely utilized for the static assessment of bridge capacity. Its origin followed the work of Kooharian [3] which introduced the limit analysis of voussoir arches, based on the main hypothesis that stone arches fail by forming pin joints. Heyman [4][5] proposed the well-known model of arch made up of a set of rigid voussoirs laid dry without any mortar, with the assumptions of material with no tensile strength but infinite compressive strength and that sliding failure cannot occur. These

hypotheses found their justifications in the facts that no tensile forces can be transmitted from one voussoir to another and stresses are usually low enough so that crushing of the material is not allowed and that friction between voussoirs is very high. The application of limit analysis to arch bridges with infinitely rigid and resistant material was analysed by Clemente *et al.* [6].

The mechanism model, with the Heiman's hypotheses, was also used to analyse the behaviour under seismic actions. The dynamic behaviour of the circular arch ring is analysed in [7], where failure conditions under horizontal acceleration are established and discussed. Clemente [8] analysed the dynamic behaviour of an arch without backfill under sinusoidal base acceleration and focused the attention on the importance of frequency content and amplitude of the input so as on the initial conditions. The analysis was limited to the first half cycle of vibration, so the impulsive behaviour during impacts was neglected.

The impact condition was addressed in [9] for the circular arch ring, in the hypotheses that the hinge locations in the four link mechanism do not vary and they reflect when the motion is inverted. Furthermore, the model does not allow initial free hinge formation and assumes two hinges at springing points [10]. The latter aspect is of interest, given that, depending on the value of the embracing angle, the thickness of the arch and the intensity of the horizontal acceleration, the four-hinge mechanism will hinge either only at one or at both springing points, as shown in [8] and confirmed by the variational method shown in [11].

From a technical point of view the comparison between the peak ground acceleration (PGA) and the minimum acceleration necessary to turn the structure into a four link mechanism can be assumed as safety criterion, even though after some oscillations, the arch may return to its natural configuration.In this way, the safety check of a stone arch under seismic actions is a static matter.

On the basis of these considerations, Clemente and Raithel [12] carried out a comprehensive numerical investigation on the limit analysis of stone voussoir arches under horizontal loadings simulating seismic actions. The model of arch subject to its self-weight and back-fill was assumed, considering both parabolic and circular shape. Four models were considered to simulate the structure - back-fill interaction. In the first one (M1) each voussoir on the left side is subjected to the inertial force due to an horizontal strip of the back-fill, and acting at its center point; the back-fill on the right tends to separate from the arch, because of its inertial forces. In the second model M2 also the right side of the arch is subjected to the inertial force due to an horizontal strip of the back-fill, and acting at its center point. In the third model each voussoir is subjected to a horizontal force proportional to the vertical loads acting on it. Finally, in the fourth model (M4) the inertial load is simulated with uniform load acting on the left half arch, whose resultant is equal to the total inertial load of structure - plus - back-fill. The influence of geometrical and loading parameters on the mechanism and the corresponding load factor was analysed. With reference to the different models considered for the seismic actions the Authors stated that the curves relative to model M1 showed load factor values always greater than the others, while the minimum values were obtained by using model M4. The simplest model, M3, for which the horizontal forces are proportional to the vertical ones showed values of the load factor intermediate between those of cases M1 and M2.

As well known, the mechanism method presents very important advantages, such as the simplicity and the speed, but its application could be limited due to the assumed hypotheses. Among these the effective value of the compression strength, which is not

infinite, even though could be very high. Relevant proposals have been done by Harvey [13], based on the definition of the thrust zone, which is of sufficient depth to carry the load at each cross-section, and Taylor and Mallinder [14] that make use of a parabolic stress-strain constitutive relationship derived from empirical observation to describe the local crushing of masonry at hinge development.

In this paper the arch of no tension material and rigid-perfect plastic behaviour with finite strength in compression is studied, already introduced in another paper of the same Authors [15], in which the limit behaviour under vertical load was studied. The analysis is limited to the onset of motion and the horizontal acceleration necessary to turn the structure into a mechanism is evaluated. The successive dynamic phase is not considered. The influence of geometrical parameters is analysed, both for circular or parabolic arch profiles. The analysis is limited to the arch, the three dimensionality of the problem, the contribution of spandrel walls and/or arch barrel-fill interaction have not considered.

2. OVERVIEW OF THE LIMIT ANALYSIS IN THE CASE OF MATERIAL WITH FINITE COMPRESSION STRENGTH

The model here used of arch of no tension material and rigid-perfect plastic behaviour with finite strength in compression equal to f_u, refers to a previous paper of the same Authors [15]. In a yielded section, subject to the axial force N acting at a distance d from the edge, stresses are uniformly distributed along a length $2d$ from the edge (Fig. 1).
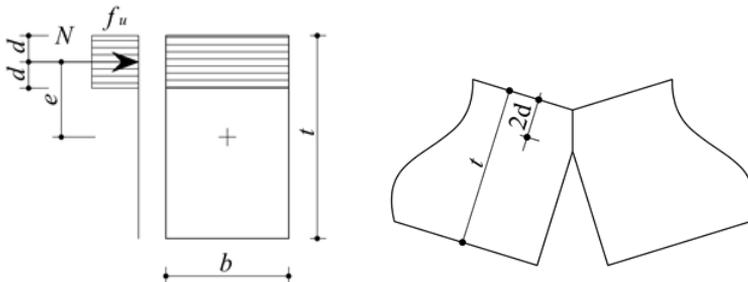

Fig. 1. *Equivalent design stress distribution and hinge.*

The yield domain of a rectangular cross-section in the plane $(\hat{e}, \hat{N})$, $\hat{e} = e/t$ and $\hat{N} = N/N_u$, with $N_u = b \cdot t \cdot f_u$, are the non-dimensional eccentricity and axial force, respectively (Fig. 2), is:

$$\hat{e} = \pm \frac{1}{2}\left(1 - \hat{N}\right) \tag{1}$$

The couples $(\hat{e}, \hat{N})$ of the limit domain correspond to limit states in which a portion of the cross-section is uniformly compressed with stress equal to f_u. Thanks to this yielded zone the two adjacent sections can rotate one with respect to the other with respect to the internal starting point of the stress diagram (Fig. 1).

The Heyman's model can be viewed as a limit case, in which $f_u \to \infty$ and $\hat{N} \to 0$, so that the two straight lines become parallel to $\hat{N}$ axis ($\hat{e} = \pm 1/2$). The collapse of the section consists in the formation of hinges at edges (Fig. 3) and failure of the arch occurs when sufficient hinges form to turn the structure into a mechanism. The internal forces acting at the hinges do not influence the equilibrium and on the point of collapse an equilibrium solution can be found, in which the resultant forces are always within the masonry.
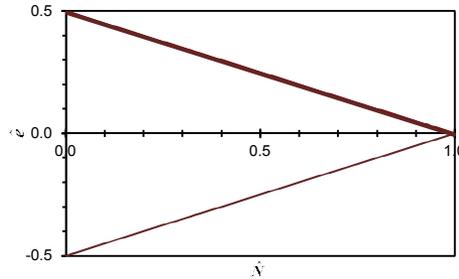

***Fig. 2.** Limit domain.*

In the hypothesis of rigid-plastic behaviour of the material with finite compression strength, the relative rotation centre between two adjacent sections on the point of collapse is coincident with the starting point of the stress diagram (Fig. 1). Therefore, the line of thrust does not pass through the hinge location and this implies some limitations to the mechanism.
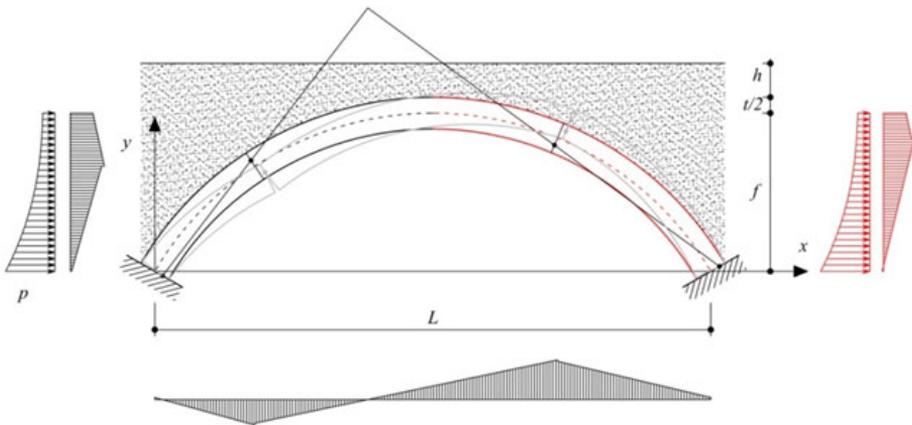

***Fig. 3.** Failure Mechanism.*

The internal forces acting at the hinges contribute to the stability of the structure. Thanks to the hypotheses about the constitutive relationship of the material and the stress distribution, all the hinges form contemporary. The uniqueness theorem ensures that this solution exists and is unique, and so is the load factor. The safe theorem states that a

structure is safe if an equilibrium solution can be found in which the couples $(\hat{e}, \hat{N})$ are always inside the limit domain. It is important reminding that the thrust line of the safe theorem does not need to be the actual thrust line: every thrust line in equilibrium with external loads, and satisfying the limit condition, if any, can be chosen to check the structure. Moreover the actual stress distribution in the cross-section is not known. In fact, the assumption about the material constitutive relationship does not allow that, but the fact that the thrust line lies within the masonry, with stresses lower than f_u, ensures that there are only compressive actions, which can be transmitted from each section to the next.

3. LIMIT ANALYSIS UNDER HORIZONTAL LOADS

For any span L, assuming the non-dimensional coordinates of the arch centre line:

$$\hat{x} = x/L \quad , \quad \hat{y}(\hat{x}) = y(x)/L \tag{2}$$

the geometrical characteristics of the arch are individualized by the non-dimensional rise ratio, the thickness ratio function and the fill depth ratio above the extrados at the crown, respectively:

$$\hat{f} = f/L \quad \hat{t}(\hat{x}) = t(x)/L \quad \hat{h} = h/L \tag{3}$$

The width b of the deck is usually assumed unitary for plane modelling. If γ_w is the weight per unit volume of the structural material, its compression strength can be defined by the non-dimensional parameter:

$$\sigma = \frac{f_u}{\gamma_w L} \tag{4}$$

The dead load w is given by the summation of:

- the arch self-weight w_w, which varies along the span even if the thickness is constant;
- the backfill weight w_b, also variable along the span, which depends also on its weight per unit volume γ_b.

The horizontal load depends on the model assumed for the structure - back-fill interaction. In this paper, the seismic horizontal load has been supposed to be proportional to the vertical load at each cross section. If a_g is the horizontal acceleration, it is:

$$p_h(x) = w(x) \cdot a_g / g \tag{5}$$

Let us consider the masonry arch in Fig. 3 and suppose that an equilibrium solution under the vertical dead loads can be found, in which the points representing the stress states on the plane $(\hat{e}, \hat{N})$ are always within the limit domain of each cross-section. When the seismic loads are put in action and are increased from zero to the collapse value, the line of thrust changes and at least four hinges form.

Collapse mechanism and the corresponding horizontal acceleration can be found by using the usual iteration procedure in which the equilibrium equation is written by means of the principle of virtual works [15]. For any assigned mechanism, the virtual work of

vertical loads w and horizontal loads p_h (with $\hat{a}_g = a_g/g = 1$) can be written using the non-dimensional expressions of the loads, respectively:

$$L_w = \gamma_w b L^3 \cdot \int_0^1 \left[\hat{w}_w(\hat{x}) + \hat{w}_b(\hat{x})\right] \cdot \eta(\hat{x}) d\hat{x} \tag{6}$$

$$L_h = \gamma_w b L^3 \int_0^1 \left[\hat{w}_w(\hat{x}) + \hat{w}_b(\hat{x})\right] \cdot \hat{\xi}(\hat{x}) d\hat{x} \tag{7}$$

where $\eta(\hat{x}) = \eta(x)/L$ and $\hat{\xi}(\hat{x}) = \xi(x)/L$ are the vertical and horizontal components of the virtual displacements, respectively. If $\Delta\varphi_i$ are the relative rotations at the n hinges $\hat{d}_i = d_i/L$ and $\hat{N}_i = 2\sigma\hat{d}_i$, then the internal work can be written:

$$L_i = 2\sum_1^n f_u d_i^2 \Delta\varphi_i = 2\gamma_w b L^3 \sum_1^n \sigma \hat{d}_i^2 \Delta\varphi_i \tag{8}$$

It is interesting to observe that when the strength of material increases indefinitely then the internal work goes to zero. If $\hat{a}_g$ is the horizontal acceleration, which turns the structure into a mechanism, the equilibrium equation is:

$$L_w + \hat{a}_g L_h = L_i \tag{9}$$

Eq. (9) gives the kinematically admissible horizontal acceleration $\hat{a}_g$ and so the load intensity associated with the assumed mechanism, from which the external reactions and the line of thrust can be found. The acceleration value $\hat{a}_g$ is the collapse acceleration only if the associated line of thrust satisfies everywhere the relation $|\hat{e}| \leq (1 - \hat{N})/2$, with the equality at a sufficient number of sections to turn the arch into a mechanism. If it is not, the procedure must be continued and in the next step the hinges must be moved to the section with the maximum exceedances.

4. NUMERICAL INVESTIGATION

The collapse horizontal acceleration has been evaluated for a circular arch with embrace angle $\beta = 125°$. The radius R of the arch is related to the span length trough the angle of embrace:

$$\hat{R} = \frac{R}{L} = \frac{1}{2 \cdot \sin(\beta/2)} \tag{10}$$

Fig. 4 shows the value of horizontal acceleration versus thickness ratio for arch ring only ($\gamma = \gamma_b/\gamma_w = 0$) or with fill($\gamma = \gamma_b/\gamma_w = 0.5$), whose mass is considered applied to arch centreline. Thickness ratios varying in the range [0.03, 0.19] have been considered.

Fig. 5 shows by red thick lines the failure domain and by coloured lines the eccentricity versus the axial force, for thickness ratios ranging from 0.06 to 0.15. Two material strength are considered; the larger the strength of material the lower the value of the non-dimensional axial force, approaching zero when the strength of material goes to infinity.
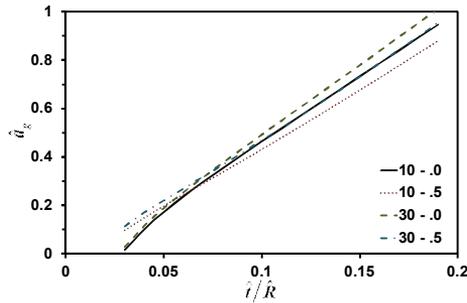
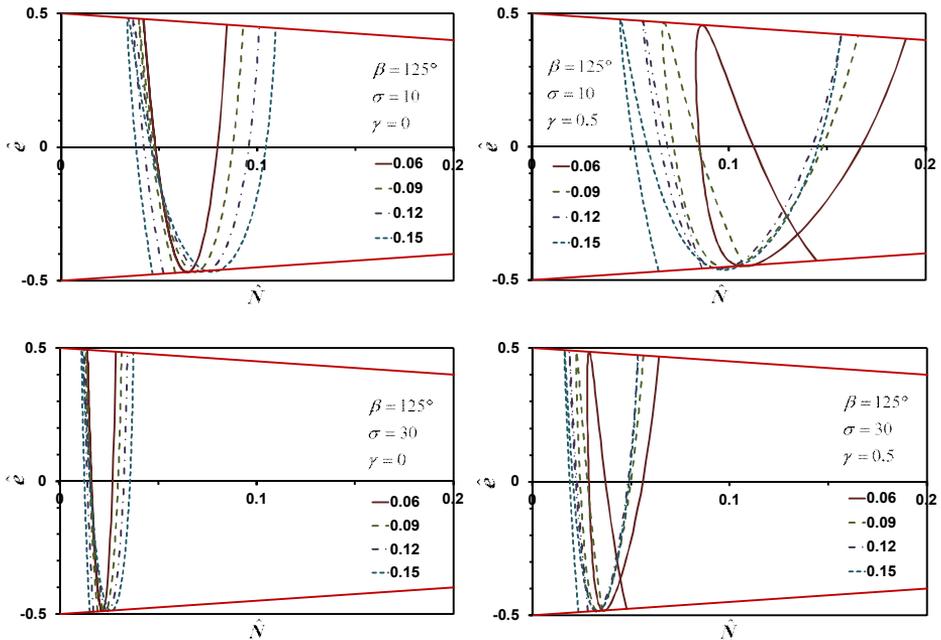**Fig. 4.** *Collapse acceleration versus thickness ratio for different cases of σ-γ.*

Fig. 5. *Values of the couples $(\hat{e}, \hat{N})$ for β=125°, for σ=30 (up) with γ=0 (up left) and γ=0.5 (up right) and for σ=30 (down) with γ=0 (down left) and γ=0.5 (down right).*

The locations $\hat{x}_{Hi} = x_{Hi}/L$ of the four hinges are shown in Fig. 6, in the cases of absence of back-fill (dashed lines) and with back-fill (continuous lines). Two cases of material strength are considered, the higher given results approaching the rigid material case treated in [8].
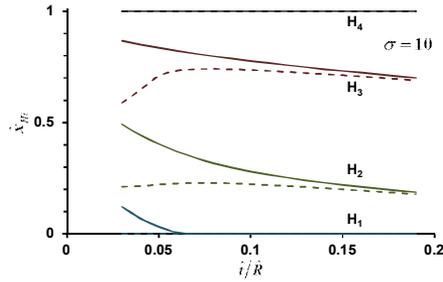
Fig. 6. *Plastic hinge location for $\sigma = 10$ with $\gamma = 0$ (dashed) and $\gamma = 0.5$ (continuous). The corresponding curves for $\sigma = 30$ are very close to these.*
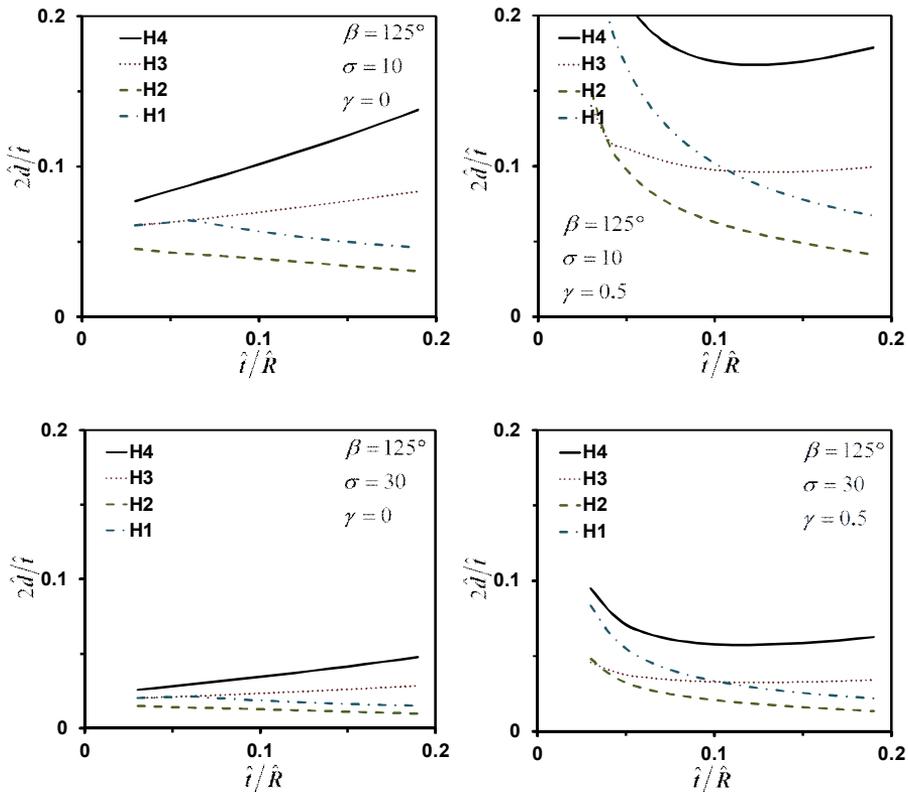

Fig. 7. *Ratio $2\hat{d}/\hat{t}$ for σ=10 (up) with γ=0 (up left) and γ=0.5 (up right) and σ=30 (down) with γ=0 (down left) and γ=0.5 (down right).*

The differences are quite low for hinge location, at least for the two considered values of strength. Figs. 7 show the depth of the plastic zones at hinges using the same

representation criteria given previously. Large differences arise when the fill weight has taken into account.

5.   CONCLUSIONS

The limit behaviour of arches made of no tension material with finite compression strength under fixed vertical loads, which include the ring self-weight and the back-fill, and variable horizontal seismic load has been analysed in this paper. The fill weight has been considered as an additional mass attached to the arch centre line. A rigid - perfect plastic model has been used for the material. The analysis was based on a non-dimensional formulation and has been applied in this paper to circular arches with or without fill but can be used for any arch profile.

The numerical investigation allowed finding out some aspects of the structural behaviour. In general the horizontal acceleration which leads to collapse increases with the thickness ratio but presents a significant lowering with the strength of the material, even in the range of practical interest. The contribution of the back-fill increases the collapse acceleration in the lower range of thickness ratio, whereas for high thickness ratio the collapse acceleration is lowered. The hinge location is scarcely affected by the strength, but the presence of the fill moves the hinges position in the lower range of thickness.

REFERENCES

[1]   PELA' L., APRILE A. and BENEDETTI A., Seismic assessment of masonry arch bridges, *Engineering Structures*, Vol. 31, No. 8, 2009, pp. 1777-1788.

[2]   DEJONG M., and OCHSENDORF J., Analysis of vaulted masonry structures subjected to horizontal ground motion, *Proceedings of Fifth International Conference on Structural Analysis of Historical Constructions*, New Delhi, 2006, pp. 973-980.

[3]   KOOHARIAN A., Limit Analysis of Voussoir (Segmental) and Concrete Arches, *Journal of the American Concrete Institute*, Vol. 24, No. 4, 1952, pp. 317-328.

[4]   HEYMAN J., The Stone Skeleton. *International Journal of Solids and Structures*, Vol. 2, No. 2, 1966, pp. 249-279.

[5]   HEYMAN J., The safety of masonry arches. *International Journal of Mechanical Sciences*, Vol. 11, No. 4, 1969, pp. 363-385.

[6]   CLEMENTE P., OCCHIUZZI A. and RAITHEL A., Limit Behavior of Stone Arch Bridges, *ASCE Journal of Structural Engineering*, Vol. 121, No. 7, 1995, pp. 1045-1050.

[7]   OPPENHEIM I.J., The Masonry Arch as a Four-Link Mechanism under Base Motion, *Earthquake Engineering and Structural Dynamics*, Vol. 21, No. 11, 1992, pp. 1005-1017.

[8]   CLEMENTE P., Introduction to dynamics of stone arches. *Earthquake Engineering and Structural Dynamics*, Vol. 27, No. 5, 1998, pp. 513-522.

[9]   DE LORENZIS L., DeJONG M. and OCHSENDORF J., Failure of masonry arches under impulse base motion, *Earthquake Engineering and Structural Dynamics*, Vol. 36, No. 14, 2007, pp. 2119-2136.

[10]   DeJONG M., DE LORENZIS L., ADAMS S. and OCHSENDORF J., Rocking Stability of Masonry Arches in Seismic Regions, *Earthquake Spectra*, Vol. 24, No. 4, 2008, pp. 847–865.

[11]   ALEXAKIS H., and MAKRIS N., Limit equilibrium analysis and the minimum thickness of circular masonry arches to withstand lateral inertial loading. *Archive of Applied Mechanics*, Vol. 84, No. 5, 2014, pp. 757-772.

[12]   CLEMENTE P. and RAITHEL A., The mechanism model in the seismic check of stone arches. In Sinopoli A. (ed) *Arch Bridges*, Balkema, Rotterdam, 1998, pp. 123-129.

[13]   HARVEY, W.J., Application of the mechanism analysis to masonry arches, *Structural Engineer* 66, 1988, pp. 77-84.

[14]   TAYLOR, N., and MALLINDER, P.A., The brittle hinge in masonry arch mechanisms. *The Structural Engineer*, 71, 1993, pp. 359-366.

[15]   CLEMENTE P. and SAITTA F., Analysis of No-Tension Material Arch Bridges With Finite Compression Strength. *Submitted*, 2016.