SAFETY FACTORS ASSESSMENT FOR IN-PLANE STABILITY OF CONCRETE FILLED STEEL TUBULAR ARCHES USING INVERSE FINITE ELEMENT RELIABILITY METHOD

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SUMMARY

Concrete filled steel tubular arch(CFST) bridges have been widely built worldwide in the past two decades. However, studies on the probability-based stability design of this type of bridges are limited. Therefore, an inverse finite element reliability method (FERM) is presented to solve the in-plane stability safety factors of CFST arches. The cooperation of self-programming and general finite element software is utilized to implement the method. In this method, a safety factor is introduced into the limit state equation. For different target reliability indices, safety factors for in-plane stability are solved based on the inverse reliability analysis. The results show that: the method is of good efficiency and applicability; loading styles and steel ratios have little effect on safety factors for the in-plane stability of CFST arches; the influences of resistance and load uncertainties on the in-plane stability factors are significant which should be concerned in design.

Keywords: *CFST arch, in-plane stability, inverse reliability, FERM, safety factor assessment.*

1. INTRODUCTION

Concrete-filled steel tubular (CFST) structures have been widely used in arch bridges over the past 20 years due to the advantages of light weight, high ultimate compressive strength, convenience of construction and good aesthetic appearance. According to incomplete statistics, more than 400 CFST arch bridges have been constructed worldwide and 300 of them are built in China [1]. Investigations on both theory and experiment of CFST arch bridges have attracted some attentions from researchers and some remarkable conclusions have been obtained [2-3].

However, the present investigations of CFST arch bridges are concentrated on the applications of new bridge types and new techniques. The gap between the rapid development of practical engineering and the slowly development of theoretical research becomes more and more serious.

Moreover, CFST arch bridges have become a member of large span bridges. For the properties of high strength and large span, the arch ribs are commonly quite slender, and the problem of stability becomes more significant. However, studies on the stability design of this type of bridges are limited, especially for the in-plane nonlinear stability.

The inverse reliability problem is presented to directly seek the value of design parameters corresponding to specified reliability levels. The problem used to be solved by the trial and error method by using a forward reliability method and interpolating the parameters at the required reliability. For the complexity of the trial and error method, the Hasofer-Lind-Rackwitz- Fiessler (HLRF) iterative algorithm was proposed by Der Kiureghian et al [5]. The efficiency and applicability of the inverse reliability method have been proved by Li and Foschi, using several examples related to the earthquake and offshore engineering [6]. The inverse reliability method has also been applied in the assessment of main cable safety factors for long-span suspension bridges and the efficiency was verified by some numerical examples [7].

In the present work, the inverse reliability method is introduced in the safety factors assessment for in-plane stability of CFST arches. Safety factors for different parameters are obtained using the inverse finite element reliability method (FERM).

2. PRINCIPLES OF INVERSE RELIABILITY ANALYSIS

The inverse reliability problem arises when one is seeking directly the value of design parameters corresponding to specified reliability levels. The inverse reliability problem is defined by the following set of equations[8].

$$\|\boldsymbol{u}\| - \beta_{\mathrm{T}} = 0 \tag{1}$$

$$\boldsymbol{u} + \frac{\|\boldsymbol{u}\|}{\|\nabla_{\boldsymbol{u}} G(\boldsymbol{u},\boldsymbol{\theta})\|} \nabla_{\boldsymbol{u}} G(\boldsymbol{u},\boldsymbol{\theta}) = \mathbf{0} \tag{2}$$

$$G(\boldsymbol{u},\boldsymbol{\theta}) = 0 \tag{3}$$

where $\boldsymbol{u}$ is the standard normal vector, β_{T} is the target reliability index, $\nabla_{\boldsymbol{u}}$ is the gradient operator with respect to $\boldsymbol{u}$ and Eq. 2 states that the solution $\boldsymbol{u}^*$ must be an origin-project point, which is the optimality condition for a fixed $\boldsymbol{\theta}$ under the condition $G(0,\boldsymbol{\theta}) > 0$.

For a target reliability index β_{T}, the inverse problem can be stated as:

Given β_{T},

Find $\bar{\theta}$ (mean value of θ) or σ_θ (standard deviation of θ),

Subjected to: $\min(\boldsymbol{u}^{\mathrm{T}}\boldsymbol{u}) = \beta_{\mathrm{T}}^2$ and $G = G(\boldsymbol{u},\theta) = 0$.

For the first order reliability method (FORM), the latter constructs a sequence of points according to the rule

$$\begin{pmatrix} \boldsymbol{u}_{k+1} \\ \boldsymbol{\theta}_{k+1} \end{pmatrix} = \begin{pmatrix} \boldsymbol{u}_{k} \\ \boldsymbol{\theta}_{k} \end{pmatrix} + \lambda_k \boldsymbol{d}_k \tag{4}$$

where, $\boldsymbol{d}_k$ is the search direction vector, λ_k is the step size.

The solution of Eq. 1, Eq. 2 and Eq. 3 is

$$\boldsymbol{u} = -\beta_{\mathrm{T}} \frac{\nabla_{\boldsymbol{u}} G(\boldsymbol{u}_k,\boldsymbol{\theta}_k)}{\|\nabla_{\boldsymbol{u}} G(\boldsymbol{u}_k,\boldsymbol{\theta}_k)\|} \tag{5}$$

$$\boldsymbol{\theta} = \boldsymbol{\theta}_k + \frac{[\nabla_{\boldsymbol{u}} G(\boldsymbol{u}_k, \boldsymbol{\theta}_k), \boldsymbol{u}_k] - G(\boldsymbol{u}_k, \boldsymbol{\theta}_k) + \beta_{\mathrm{T}} \left\| \nabla_{\boldsymbol{u}} G(\boldsymbol{u}_k, \boldsymbol{\theta}_k) \right\|}{\nabla_{\boldsymbol{\theta}} G(\boldsymbol{u}_k, \boldsymbol{\theta}_k)} \tag{6}$$

Then $\boldsymbol{d}_k$ can be expressed as

$$\boldsymbol{d}_k = \begin{pmatrix} -\beta_{\mathrm{T}} \dfrac{\nabla_{\boldsymbol{u}} G(\boldsymbol{u}_k, \boldsymbol{\theta}_k)}{\left\| \nabla_{\boldsymbol{u}} G(\boldsymbol{u}_k, \boldsymbol{\theta}_k) \right\|} - \boldsymbol{u}_k \\ \dfrac{[\nabla_{\boldsymbol{u}} G(\boldsymbol{u}_k, \boldsymbol{\theta}_k), \boldsymbol{u}_k] - G(\boldsymbol{u}_k, \boldsymbol{\theta}_k) + \beta_{\mathrm{T}} \left\| \nabla_{\boldsymbol{u}} G(\boldsymbol{u}_k, \boldsymbol{\theta}_k) \right\|}{\nabla_{\boldsymbol{\theta}} G(\boldsymbol{u}_k, \boldsymbol{\theta}_k)} \end{pmatrix} \tag{7}$$

The convergence criterion used in the inverse reliability analysis is

$$\frac{\left(\left\| \boldsymbol{u}_{k+1} - \boldsymbol{u}_k \right\|^2 + \left| \boldsymbol{\theta}_{k+1} - \boldsymbol{\theta}_{k+1} \right|^2 \right)^{1/2}}{\left(\left\| \boldsymbol{u}_{k+1} \right\|^2 + \left| \boldsymbol{\theta}_{k+1} \right|^2 \right)^{1/2}} \le 10^{-3} \tag{8}$$

3. LIMIT STATE EXPRESSIONS OF IN-PLANE STABILITY DESIGN

The limit state expression for stability can be expressed as

$$R / K - S > 0 \tag{9}$$

where R is the structural resistance, K is the stability safety factor and S is the load effect.

Then limit state expressions for elastic theory and nonlinear theory can be respectively expressed as

$$P_{\mathrm{cr}} / K_1 - S > 0 \tag{10}$$

$$P_{\mathrm{u}} / K_2 - S > 0 \tag{11}$$

where P_{cr} is the elastic stability bearing capacity, P_{u} is the nonlinear stability bearing capacity, K_1 is the safety factor for elastic theory, K_2 is the safety factor for nonlinear theory.

Consequently, the basic form of limit state equation for inverse reliability analysis can be expressed as

$$G(\boldsymbol{u}, y_{\mathrm{cap}}) = g(\boldsymbol{x}, y_{\mathrm{cap}}) = y_{\mathrm{cap}} - r(\boldsymbol{x}) = y_{\mathrm{cap}} - R(\boldsymbol{u}) \tag{12}$$

where y_{cap} is a deterministic design parameter.

By introducing the safety factors, the typical form of limit state equation for inverse reliability analysis is

$$G(\boldsymbol{u}, \lambda_{\min}) = g(\boldsymbol{x}, \lambda_{\min}) = r(\boldsymbol{x}) - \lambda_{\min} = R(\boldsymbol{u}) - \lambda_{\min} \tag{13}$$

Then Eq. 5 can be rewritten as

$$\boldsymbol{u} = -\beta_{\mathrm{T}} \frac{\nabla_{\boldsymbol{u}} R(\boldsymbol{u}_k, \boldsymbol{\theta}_k)}{\left\| \nabla_{\boldsymbol{u}} R(\boldsymbol{u}_k, \boldsymbol{\theta}_k) \right\|} \tag{14}$$

For the explicit form of limit state functions (LSFs), the values and gradients of LSF could be easily obtained. While for the implicit form of LSFs, the finite element analysis is utilized. The gradients are calculated by the central difference method:

$$\frac{\partial g}{\partial y} \approx \frac{g(y+\Delta y)-g(y-\Delta y)}{2\Delta y} \tag{15}$$

4. PROBABILITY MODELING

It is inevitable that the structural resistance is affected by various uncertainties, such as material properties, geometric parameters and calculation modes. For the reliability analysis of arch stability, the statistics of the basic random variables (RVs) are listed in Tab. 1.

Table 1. *Statistics of basic RVs [9-12].*

RV	Distribution Type	COV
f_{cu}	Lognormal	0.13
f_y	Lognormal	0.083
E_c	Lognormal	0.10
E_s	Lognormal	0.06
D	Normal	0.0135
T	Normal	0.035
y_0	Normal	0.5
P	Normal	0.08

The meanings of the parameters in Tab. 1 are: f_{cu} is the concrete compressive cube strength, f_y is the yield strength of steel tube, E_c means the elastic modulus of concrete, E_s means the elastic modulus of steel, D is the outer diameter of steel tube, t is the wall thickness of steel tube, y_0 is the initial geometric imperfection of arches and P is the load effect. COV is the short for coefficient of variation.

5. PROCEDURES OF INVERSE RELIABILITY METHOD

While searching the design points in the inverse reliability method, the values of LSFs and its gradients are calculated in every iterative step via finite element analysis. Therefore, the efficiencies of the inverse reliability method rely on the efficiencies of finite element analysis and the times of iteration. For nonlinear finite element analysis, the general finite element software ABAQUS is utilized, and the cooperation of MATLAB and ABAQUS is realized via the application programming interface (API) in MATLAB. The procedure of FERM is illustrated in Fig. 1.

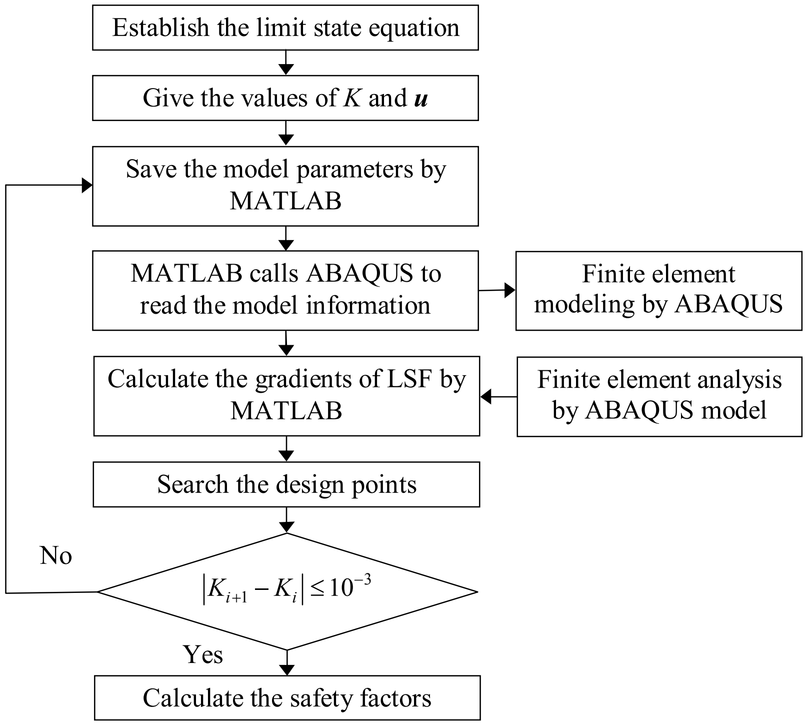

Fig. 1. *Flow chart of inverse reliability method.*

6. CASE STUDIES

6.1. Model parameters

Two CFST parabolic arch models for different loading styles are selected as illustrated examples, as shown in Fig. 2 and Fig. 3. The span of the arch is 4.6 m and the rise-to-span ratio is 1/3. The outer diameter of the steel tube is 76 mm and the wall-thickness is 3.792 mm. The elastic modulus of the concrete is 36.8 GPa, and the elastic modulus of the steel is 206 GPa.

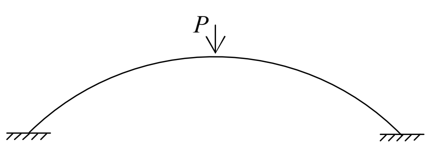

Fig. 2. *Loading at L/2.*

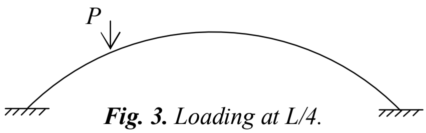

Fig. 3. *Loading at L/4.*

6.2. Results

6.2.1. Finite element analysis

For the two loading styles, comparisons of the experimental results and the mathematical results are listed in Tab. 2.

Table 2. *Comparisons of experimental results and mathematical results.*

Loading styles	**Experiment/kN**	**Finite element/kN**		
		ABAQUS	**ANSYS**	**Self-programming**
Loading at $L/4$	31.90	32.53	32.62	32.80
Loading at $L/2$	42.07	46.23	46.46	47.90

The results show that: compared with ANSYS and the self-programming, the results of in-pane ultimate stability bearing capacity calculated by ABAQSUS agree well with the experiments.

6.2.2. Inverse reliability analysis

K_2 for different loading styles with considering resistance uncertainties are summarized in Tab. 3.

Table 3. *K_2 for different loading styles.*

Target reliability index	**Loading styles**	
	Loading at $L/4$	**Loading at $L/2$**
3.7	1.35	1.34
4.2	1.40	1.39
4.7	1.46	1.45
5.2	1.52	1.50

The results show that: K_2 varies from 1.34~1.52 for different target reliability indices and loading styles have little effect on K_2.

K_2 for different steel ratios with considering resistance uncertainties are summarized in Tab. 4.

Table 4. *K_2 for different steel ratios.*

Target reliability index	Steel ratios			
	5%	**10%**	**15%**	**20%**
3.7	1.37	1.35	1.35	1.35
4.2	1.43	1.41	1.40	1.40
4.7	1.50	1.47	1.46	1.46
5.2	1.56	1.53	1.52	1.52

The results show that: K_2 varies from 1.35~1.56 for different target reliability indices and steel ratios have little effect on K_2.

Comparisons of K_2 and K_2' are shown in Fig. 4. K_2 is the safety factor with considering resistance uncertainties while K_2' is the safety factor with considering both resistance uncertainties and load uncertainties.

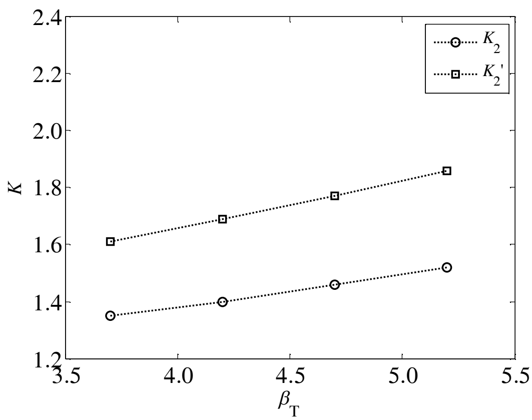

Fig. 4. *Comparisons of K_2 and K_2'.*

The results show that: K_2' varies from 1.61~1.86 for different target reliability indices which are higher than K_2.

7. CONCLUSIONS

The inverse finite element reliability method is presented to solve the safety factors for in-plane stability of CFST arches. The cooperation of programming and general finite element software is utilized to implement the method. For different design parameters, in-plane stability safety factors with and without considering load uncertainties are obtained based on arch models.

The results show that: the method above is of good efficiency and applicability. Loading styles and steel ratios have little effect on in-plane nonlinear stability safety factors of

CFST arches. For the parameters discussed in this paper, K_2 varies from 1.34~1.56 while K_2' varies from 1.61~1.86. Therefore, effects of both resistance uncertainties and load uncertainties on in-plane stability factors are significant which should be concerned in design.

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